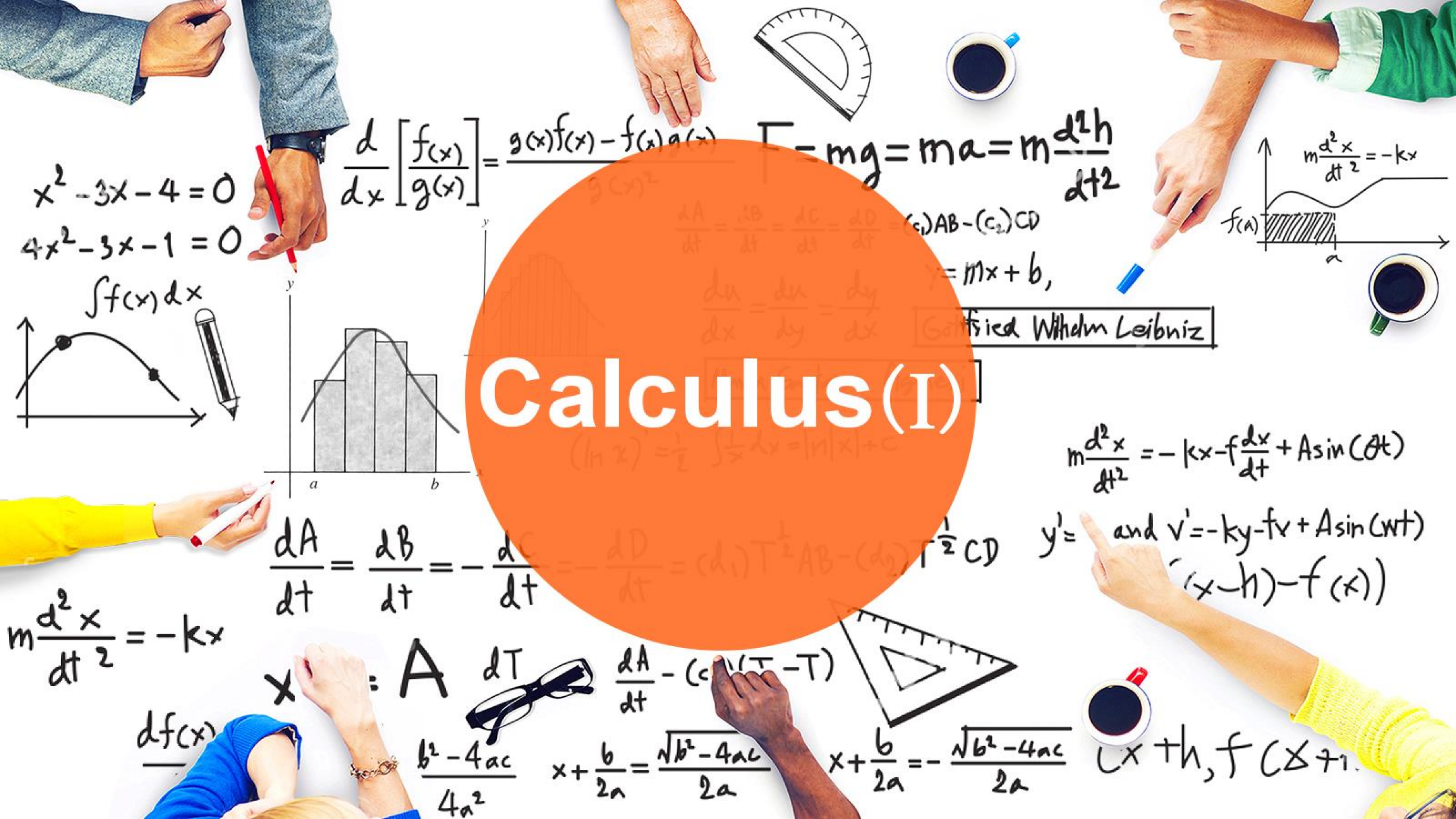
Calculus(Ⅰ)



x² - 3x - 4 = 0
4x² - 3x - 1 = 0
∫f(x) dx
F = mg = ma = m d²h/dt²
Wilhelm Leibniz
m d²x/dt² = -kx
m d²x/dt² = -kx - f dx/dt + Asin(θt)
x + b/2a = √(b² - 4ac)/2a
x + b/2a = - √(b² - 4ac)/2a

2.3 Rules for Finding Derivatives

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How to find the derivatives of the following expressions?

$$[f(x)+g(x)]',\ [f(x)-g(x)]',\ [f(x)\cdot g(x)]',\ [f(x)/g(x)]'$$

addition

subtraction

multiplication

division

According to the definition of derivative.

Rules for Finding Derivatives

Th A $C' = 0, \quad (C \text{ is a constant})$

Th B $x' = 1$

Th C $(x^n)' = nx^{n-1}$

Th D $(kf(x))' = k \cdot f'(x)$

Th E $[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

Th F $[f(x) \cdot g(x)]' = f'(x)g(x) + f(x)g'(x)$

Th G $[f(x)/g(x)]' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (g(x) \neq 0)$

Rules for Finding Derivatives

According to Th G,

$$[f(x)/g(x)]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (g(x) \neq 0)$$

we have $\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{g^2(x)}$

Corollary: $(f+g+h)' = f' + g' + h'$

$$(fgh)' = f'gh + fg'h + fgh'$$

Example 1

Find the derivative of $y = x^3 - 2x^2 + \sin x$

$$y' = 3x^2 - 4x + \cos x.$$

Example 2

Find the derivative of $y = \sin 2x \cdot \ln x$

$\because y = 2\sin x \cdot \cos x \cdot \ln x$

$y' = 2\cos x \cdot \cos x \cdot \ln x + 2\sin x \cdot (-\sin x) \cdot \ln x + 2\sin x \cdot \cos x \cdot \frac{1}{x}$

$= 2\cos 2x \ln x + \frac{1}{x}\sin 2x.$

Example 3

Find the derivative of $y=\dfrac{x-1}{x+1}$ $\qquad \left(\dfrac{1}{g(x)}\right)'=\dfrac{-g'(x)}{g^2(x)}$

Method 1

$$y'=\frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}=\frac{2}{(x+1)^2}.$$

Method 2

$$y=\frac{x-1}{x+1}=1-\frac{2}{x+1},$$

$$y'=(1)'-(\frac{2}{x+1})'=-2\frac{-1}{(1+x)^2}=\frac{2}{(x+1)^2}.$$

Example 4

Find $D_x y$ if $y = \frac{2}{x^4+1} + \frac{3}{x}$.

$$D_x y = D_x\left(\frac{2}{x^4+1}\right) + D_x\left(\frac{3}{x}\right)$$

$$= \frac{(x^4+1)D_x(2) - 2D_x(x^4+1)}{(x^4+1)^2} + \frac{xD_x(3) - 3D_x(x)}{x^2}$$

$$= \frac{(x^4+1)(0) - (2)(4x^3)}{(x^4+1)^2} + \frac{(x)(0) - (3)(1)}{x^2}$$

$$= \frac{-8x^3}{(x^4+1)^2} - \frac{3}{x^2}$$

Example 5

Show that the Power Rule holds for negative integral exponents; that is, $D_x(x^{-n}) = -nx^{-n-1}$.

$$D_x(x^{-n}) = D_x\left(\frac{1}{x^n}\right)$$

$$= \frac{x^n \cdot 0 - 1 \cdot nx^{n-1}}{x^{2n}}$$

$$= \frac{-nx^{n-1}}{x^{2n}}$$

$$= -nx^{-n-1}$$

Summary

Th E

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

Th F

$$[f(x) \cdot g(x)]' = f'(x)g(x) + f(x)g'(x)$$

Th G

$$[f(x)/g(x)]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (g(x) \neq 0)$$

Questions and Answers

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Find the derivative of $y = \tan x$

$$y' = (\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

Namely, $(\tan x)' = \sec^2 x.$ $(\cot x)' = -\csc^2 x.$

Similarly, $(\sec x)' = \sec x \cdot \tan x.$ $(\csc x)' = -\csc x \cdot \cot x.$

Questions and Answers

Find the derivative of $(3x^2-5)(2x^4-x)$ by use of the Product Rule

Check the answer by doing the problem in a different way

$$\left[(3x^2-5)(2x^4-x)\right]' = (3x^2-5)'(2x^4-x)+(3x^2-5)(2x^4-x)'$$

$$= (6x)(2x^4-x)+(3x^2-5)(8x^3-1)$$

$$= 24x^5-3x^2-40x^3+5+12x^5-6x^2$$

$$= \boxed{36x^5-40x^3-9x^2+5}$$

To check, we first multiply and then take the derivative.

$$\boxed{(3x^2-5)(2x^4-x) = 6x^6-10x^4-3x^3+5x}$$

Thus, $\left[(3x^2-5)(2x^4-x)\right]' = (6x^6)'-(10x^4)'-(3x^3)'+(5x)' = \boxed{36x^5-40x^3-9x^2+5}$

Questions and Answers

Find $\left(\dfrac{3x-5}{x^2+7}\right)'$

$$\left[\frac{3x-5}{x^2+7}\right]' = \frac{(x^2+7)(3x-5)' - (3x-5)(x^2+7)'}{(x^2+7)^2}$$

$$= \frac{(x^2+7)(3) - (3x-5)(2x)}{(x^2+7)^2}$$

$$= \frac{-3x^2+10x+21}{(x^2+7)^2}.$$

Rules for Finding Derivatives